

# Math 503 - Functional Analysis

(Analysis C)

**Blue Book description:** Banach spaces and Hilbert spaces. Dual spaces. Linear operators. Hahn-Banach theorem, open mapping theorem, uniform boundedness principle. Weak derivatives and Sobolev spaces. Semigroups of linear operators. Applications to linear differential equations.

## Course Objectives

The course will develop the theory needed to treat linear integral and differential equations, within the framework of infinite-dimensional linear algebra. Applications to some classical equations will be presented.

## Prerequisites

- Math 501 (Analysis A) Lebesgue measure theory.
- Math 535 (Algebra A) Introduction to linear Algebra.

## Course Topics

1. Normed spaces. Banach spaces. Linear operators. Examples.
2. Spaces of bounded linear operators. The uniform boundedness principle and the open mapping theorem.
3. Bounded linear functionals. Dual spaces. The Hahn-Banach extension theorem. Separation of convex sets.
4. Spaces of continuous functions. Ascoli's theorem, Stone-Weierstrass' theorem. The space  $\mathcal{C}^{0,\gamma}$  of Hölder continuous functions and the space  $\mathcal{C}^k$  of  $k$ -times differentiable functions.
5. Hilbert spaces. Perpendicular projections. Orthonormal bases. Self-adjoint operators.
6. Compact operators on a Hilbert space. Fredholm's alternative. Spectrum and eigenfunctions of a compact, self-adjoint operator. Applications to Sturm-Liouville boundary value problems.
7. Weak derivatives. Sobolev spaces. Approximation by smooth functions. Regularity of functions  $f \in W^{k,p}$ . Traces, imbedding theorems. The Hilbert-Sobolev spaces  $H^k$ . Applications to linear elliptic equations.
8. Semigroups of linear operators; generators, resolvents. Application to linear parabolic and

hyperbolic PDE's.

## References

- [M] Milan Miklavcic, *Applied Functional Analysis and Partial Differential Equations*. World Scientific Publishing Co., River Edge, NJ, 1998.
- [Ru] Walter Rudin, *Functional Analysis*. Second Edition. McGraw-Hill, 1991.
- [Y] K. Yosida, *Functional Analysis*. Springer-Verlag, Berlin, 1995.