MATH 513 - Partial Differential Equations I

Topics covered in the course (Fall 2004)

1. **Basic concepts.** Classical solutions, the Cauchy problem, the boundary value problem. Well posedness.

2. **The linear transport equation.** The initial value problem, non-homogeneous problems.

3. **The Laplace equation.** Physical interpretation, fundamental solution, Poisson's equation. Mean-value formulas, maximum principles, uniqueness of solutions. Estimates on derivatives, Liouville's theorem. Harnack's inequality. Green's function. Uniqueness of solutions by the energy method, Dirichlet's principle.

4. **The heat equation.** Physical interpretation, fundamental solution, the initial value problem on the whole space $\mathbb{R}^n$, the non-homegeneous problem. Maximum principles, uniqueness of solutions on bounded and unbounded domains. Energy methods, backward uniqueness.

5. **The wave equation.** Physical interpretation, solutions on the real line by D'Alembert's formula, solution on the half line by reflection. The method of spherical means for solutions in $\mathbb{R}^2$ and in $\mathbb{R}^3$. Energy methods, finite propagation speed.

6. **Nonlinear first order PDE's**. The method of characteristics for semilinear, quasilinear, and fully nonlinear first order equations. Local existence of solutions to the initial value problem for the equation $u_t + f(x, u)\nabla_x u = 0$.

7 **Hamilton-Jacobi equations.** The standard problem in the Calculus of Variations, necessary condition for optimality: the Euler-Lagrange equations. Hamilton's equations. The Legendre transform, the Hamilton-Jacobi PDE, the Hopf-Lax formula.

8. **Transform methods.** The Fourier transform, definition and basic properties. Applications to linear PDEs with constant coefficients. $\mathbf{L}^2$ stability of solutions.

Reference: L. C. Evans, *Partial Differential Equations*, American Mathematical Society, Graduate Studies in Mathematics Vol.19, Providence 1998.