

# Math 528 - Differentiable Manifolds

**Blue Book Summary:** Smooth manifolds, smooth maps, Sard's theorem. The tangent bundle, vector fields, differential forms, integration on manifolds. Foliations. De Rham cohomology; simple applications. Lie groups, smooth actions, quotient spaces, examples.

## Syllabus

1. Review of differentiable maps on  $\mathbb{R}^n$ , including the inverse function theorem (for  $\mathcal{C}^\infty$  maps). Diffeomorphisms.
2. Topological manifolds, charts, atlases. Smooth atlases. Smooth manifolds. Smooth maps between manifolds. Examples.
3. Existence of smooth partitions of unity. The tangent space to a manifold at a point; the tangent map to a smooth map.
4. Submanifolds. The inverse image of a regular value is a submanifold. Sard's theorem.
5. Vector bundles: the tangent and cotangent bundles. Operations ("continuous functors") producing new vector bundles from old. Bundle maps. Pull-back of a vector bundle.
6. Vector fields, as sections of the tangent bundle and as differential operators. The Lie bracket and the Jacobi identity. The flow of a vector field. The Lie derivative.
7. Differential forms. Orientation. Integration on manifolds.
8. The exterior derivative: relation to the Lie bracket. The identity  $d^2 = 0$ . Stokes' theorem.
9. Foliations. Frobenius' integrability theorem. Relation to classical problems in PDE.
10. De Rham cohomology. Homotopy invariance of de Rham cohomology. Simple computations including the cohomology of spheres, the general case of the Brouwer fixed point theorem.
11. Lie groups and their Lie algebras: simple examples. Smooth actions of Lie groups on manifolds: necessary and sufficient condition for the quotient space to be a manifold. Examples, including Grassmannians and Stiefel manifolds.

## References

- [A] Thierry Aubin, *A Course in Differential Geometry*. American Mathematical Society, 2001.
- [B] W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*. Second edition. Academic Press, Inc., Orlando, FL, 1986.

[L] Serge Lang, *Fundamentals of Differential Geometry*. Graduate Texts in Mathematics, Springer-Verlag, 1999.