Math 535 - Linear Algebra

(Algebra A)

Blue Book description: Vector spaces. Linear transformations. Bilinear forms. Canonical forms for linear transformations.

Course Objectives

This course reviews undergraduate linear algebra and proceeds to more advanced topics. Its purpose is to provide a solid understanding of linear algebra of the sort needed throughout graduate mathematics.

Syllabus

1. **Vector spaces.** Fields. Vector spaces. Subspaces. Spanning sets. Linearly independent sets. Bases. Dimension.

2. **Linear transformations.** Kernel and image. Matrices. Direct sums and quotients. Correspondence between linear transformations and matrices. Matrix operations. Standard homomorphism theorems. Rank-nullity theorem.

3. **Inner products and quadratic forms.** Bilinear functions. Sesquilinear functions. Orthogonal sets and orthonormal sets. Norms. Schwarz' inequality and Bessel's inequality. Adjoints. Self-adjoint, normal and unitary matrices. Orthogonal projections and orthogonal complements. The Gram-Schmidt orthogonalization process. Parseval's inequality. Diagonalizability of normal operators. Quadratic forms over the real numbers. Signature. Sylvester's law. Positive (semi)definiteness. The spectral theorem.

4. **Theory of endomorphisms of a finite-dimensional vector space.** Determinants. Cramer's rule. Multiplicative property of the determinant. Minimal and characteristic polynomial. Cayley-Hamilton theorem. Modules over a principal ideal domain. Primary decomposition. Application to finitely generated abelian groups. If F is a field, $F[x]$ is a principal ideal domain. Rational and Jordan canonical forms.

References

[DF] D. S. Dummit and R. M. Foote, *Abstract Algebra*, 3rd ed., J. Wiley, 2004.

[K] A. W. Knapp, *Basic Algebra*, Birkhauser, Boston, 2006.

[L] S. Lang, *Linear Algebra.* Reprint of the third edition. Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1989.