Math 536 - Abstract Algebra

(Algebra B)

Blue Book description: Groups. Sylow's theorems. Rings. Ideals, unique factorization domains. Finitely generated modules. Fields. Algebraic and transcendental field extensions, Galois theory.

Course Objectives

This course covers fundamental concepts, needed toward the study of advanced areas in abstract algebra. The nineteenth-century triumphs of Galois theory described here remain beautiful and inspiring.

Course Topics

1. Groups. Subgroups. Cosets. Normal subgroups. Quotient groups. Group homomorphisms. Examples: symmetric groups, alternating groups, dihedral groups, general linear groups, special linear groups, orthogonal groups, and unitary groups. Homomorphism theorems. Cyclic groups.

2. Group actions. Orbits. Stabilizers. Sylow theorems and elementary consequences for simple or non-abelian groups of small order.

3. Further concepts. Direct products. Direct sums. Semi-direct products. Nilpotent and solvable groups.

4. Rings. Ideals. Quotient rings. Ring homomorphisms. Homomorphism theorems. Prime ideals. Maximal ideals. Polynomial rings. Integral domains and fields of fractions. Unique factorization domains. Euclidean domains. Principal ideal domains. Gauss' lemma. Eisenstein criterion.

5. Finitely generated modules over principal ideal domains.

6. Field extensions. Algebraic and transcendental elements. Degree of an extension. Degree of an (algebraic) element. Simple field extensions. Algebraic numbers. Construction and uniqueness of field extensions up to isomorphism. Splitting fields. Algebraic closure.

7. Finite fields. Existence and uniqueness.

References

[J] N. Jacobson, *Basic Algebra*, 2 vols, 2nd ed. Dover Reprint of W. H. Freeman and Company, San Francisco, 1985 edition.

[L] S. Lang, *Algebra*, Addison-Wesley, Reading, MA, 3rd ed. Springer-Verlag, New York, 2002.

[R] J. J. Rotman, *Algebra*, Prentice-Hall, 2002.

[W] B. L. van der Waerden, *Modern Algebra*, various translated editions.