Math 582
Introduction to C*-Algebra Theory

Description

Basic properties of C*-algebras, representation theory, group C*-algebras and crossed products, tensor products, nuclearity and exactness. Other topics as time permits.

Course Objectives

This is one of three courses whose purpose is to prepare students interested in operator algebra theory for research in that area. The course will develop the basic theory of C*-algebras, advancing as far as the theory of nuclear and exact C*-algebras. It will include a comprehensive study of examples.

Syllabus

1. **Review of Basic Hilbert Space Theory.** Hilbert spaces and projection operators. Adjoint and norm. Unitary operators, isometries and partial isometries. Compact operators and Toeplitz operators.

2. **Commutative Banach Algebras.** Spectrum and spectral radius formula. Gelfand transform. Definition and characterization of commutative C*-algebras. Functional calculus.

3. **C*-Algebra Fundamentals.** Basic definitions. Morphisms between C*-algebras, ideals and quotients. States, representations and the GNS construction.

4. **Approximately finite-dimensional algebras.** UHF algebras and Glimm's theorem. AF algebras and Bratteli diagrams. Classification theory for AF algebras.

5. **Group C*-algebras.** C*-algebras of locally compact groups. Crossed product algebras. Amenability. Irrational rotation algebras and other examples.

6. **Approximation properties.** Tensor products, exactness and nuclearity.

7. **Other Topics (As Time Permits).** Rapid decay property. Cuntz algebras. Extensions. Completely positive mappings.

Prerequisites

Basic familiarity with functional analysis, advancing as far as the spectral theorem for compact self-adjoint operators on Hilbert space, will be assumed.

Text

There will be no text, but a list of recommended books will be provided.