

# Math 597 - Topics in Number Theory

(topical course, Spring 2011)

Rather than concentrate on a single topic, this course will survey several different topics within number theory and discuss some recent work in some of these areas. Contents will include

- 1. Group cohomology.** The category of  $G$ -modules. Induced modules. Injective  $G$ -modules. Inflation-restriction. Cup-products. Definition of homology groups. The Tate groups, cohomology of finite cyclic groups. Tate's theorem. The cohomology of profinite groups.
- 2. Brauer groups.** Semisimple algebras. Semisimple modules. Tensor products of algebras. Centralizers in tensor products. The Noether-Skolem theorem. Definition of the Brauer group. Extension of the base field. Maximal subfields. The Brauer group of a non-archimedean local field. The invariant map. The Brauer group of the real numbers. Brauer groups and  $K$ -theory. The Brauer group of a global field.
- 3. The Hasse-principle and its failures.** Norms for cyclic extensions. Hasse Norm Theorem. Hasse-Minkowski Theorem. Failure of the Hasse principle for cubic forms.
- 4. Undecidability in Number Theory.** Statement of Hilbert's Tenth Problem. Diophantine sets. Recursive and recursively enumerable sets. Reductions. Diophantine models. Undecidability for rings of integers. Using solutions to Pell's equation and elliptic curves of rank one. Moret-Bailly's theorem for function fields of curves over formally real fields.